Summary

We present a novel technique for the determination of resistivity structures associated with arbitrary surface topography. The approach represents a triple-grid dc resistivity inversion technique which is based on unstructured tetrahedral meshes and a finite element forward operator applied to the secondary potential. We use a Gauss-Newton method with inexact line search to fit the data within error bounds. A global regularization scheme using special smoothness constraints is applied. The regularization parameter compromising data misfit and model roughness is determined by an L-curve method and finally evaluated by the discrepancy principle.

We apply our technique to synthetic data from a burial mound to demonstrate its functionality and efficiency. A resolution-dependent parameterization helps to keep the inverse problem small to cope with memory limitations of today's standard PC's. Thus, the approach can be applied to large-scale 3d problems. As a byproduct of the primary potential calculation we obtain a quantification of the topography effect and the corresponding configuration factors. The latter are used for data correction to prevent the reconstruction process from topography induced artefacts.

Introduction

There are approaches for the three-dimensional reconstruction of resistivity structures on the basis of direct current resistivity measurements. Most of them represent Gauss-Newton methods to iteratively minimizing the misfit between data and the model response (Park and Van, 1991). In every iteration step the forward problem has to be solved to simulate the response of the data. Most of the approaches use finite-difference (Zhang et al., 1995) or finite element (Pain et al., 2003; Sasaki, 1994) techniques. With the latter it is possible to describe geometry. However, most approaches are restricted to structured grids of block-orientation.

With unstructured tetrahedral geometrical constraints are met most flexibly. Thus meshes with locally varying discretization density can be created. This is useful for a the parameterization of the ground as well as for the forward calculation. On the one hand, the parameterization grid has to be relatively crude to limit the degrees of freedom and the ill-posedness of the inverse problem. On the other hand, the forward calculation requires a very fine grid to provide sufficiently accurate results. Therefore, a transformation is required transferring the parameter grid into the forward modeling grid. Often, the forward calculation is the most time-consuming part of the inversion. Methods as the singularity removal technique (Lowry et al., 1989) contribute to reduce computational costs in this stage.

However, the primary potentials are needed for such a procedure. If topography is present, they can only be assessed by modelling on a highly refined grid. These considerations lead to the triple-grid technique as presented in the following (see Figure 1).
The primary field defines the cells whose resistivity has to be detected. The element sizes depend on the physical resolution. The forward grid is obtained by a global refinement and prolongation to satisfy accurate simulation results. In every iteration the forward calculation is carried out using the singularity removal technique. The primary potentials needed for this have to be obtained by simulation once at the beginning of the inversion. To approximate the singular potential well, the primary grid is highly refined at the electrodes.

With the primary potential, which is obtained by a homogeneous conductivity, configuration factors can be calculated. By dividing the real configuration factors by the (wrong) flat-earth configuration factors we define the topography factor. It reveals the pure effect of the topography and can be used to correct the original data (Fox et al., 1980).

Figure 2 shows the inversion scheme. The three meshes are created from the electrode positions and the topography. By the primary potentials the configuration factors are calculated and used to correct the data. Special smoothness constraints are created to restrict the model roughness globally. The computation of the sensitivities is based on the reciprocity principle using the calculated potentials and the stiffness matrix entries. An inexact line search is applied to accelerate convergence. Another important point is the weighting of the data which ensures fitting the data within error bounds.

A synthetic example

The following example is descended from the investigation of a burial mound. It illustrates the flexibility of handling a complicated geometry. The topography is built up by intersection of an ellipsoid with a plane. Its footprint is an ellipse of 20x15m extent, the top of the mound is located 5m above the ground level. Consequently, the steep topography, particularly at the foot of the slope, affects the direct current measurements severely and cannot be neglected in the inversion process.
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9 profiles in x-direction and 13 profiles in y-direction are placed. The electrode distance is 1m on all profiles, the profile distance 2m. In summary 487 electrodes are used. The mound itself is supposed to have a resistivity of 200Ωm whereas for the underground 100Ωm are assumed. Additionally, a cavity inside the mound is considered. It has an ellipsoidal shape with half-axes of 1.5, 1.0 and 0.5m in x, y and z direction, respectively. Its center is located 2.5m below the top of the mound.

Dipole-dipole configurations are considered on all profiles. To counteract the growing configuration factor and redundancy, the dipole lengths are enlarged with increasing separation factor n. Thus a total of 6943 single data was simulated and noisified.

Figure 4: Data with flat-earth configuration factors (left), topography effect (center) and data with real configuration factors (right)

Exemplary a central profile at y=10m is shown in Figure 4. The data with the wrong configuration factors show mainly topographical effects. By the topographical correction the data show the resistivity of the underground, the mound and the high-resistive anomaly of the cavity.

The meshes are created by the free mesh generator Tetgen (Si, 2003). In the parameter grid 13109 elements or degrees of freedom are contained. The secondary grid obtained 170040 elements and the primary grid 1055157 elements, respectively. Whereas the system of equations for the primary potentials has to be solved iteratively by preconditioned conjugate gradients, the secondary grid equations can be solved by direct multifrontal methods due to its reduced size. Once the Cholesky factor is determined solution for the individual sources are easily obtained by backsubstitution. Thus the 487 single simulations can be carried out in minutes instead of hours on a standard pc.

Within 5 iterations the data misfit has been diminished from a chi-squared value of 117.0 down to 1.1 which corresponds to fitting the data within error bounds. The regularization parameter has been chosen by the L-curve method. Figure 5 shows the inversion result. Clearly the lower boundary of the mound can be distinguished from the underground resistivity. In the center the assumed cavity is indicated by the high-resistive iso-body.

Figure 5: Inversion result of the synthetical data set, the isovalue is 300Ωm.

The inversion scheme is also successfully applied to field data.
Conclusions

An inversion strategy for the reconstruction of conductivity from DC measurements for arbitrary topography was presented. It is an enhancement of existing techniques and combines the fast convergence of regularized Gauss-Newton methods with accurate finite element forward calculations. It uses unstructured tetrahedral grids in order to describe the topography of the measurement area with high accuracy. Special attention is payed to an efficient forward simulation using the singularity removal technique. Therefore we split up the forward process into two parts: the time-intensive calculation of the primary potential and the fast calculation on the moderate secondary field grid.

The strategy could be applied to noisified synthetic data as well as to field data. For both examples reasonable results could be obtained within short time. Compared with classical approaches the run time is strongly decreased by the singularity removal technique and the use of fast direct equation solvers. In addition, the use of unstructured grids provides a resolution-dependent parameterization, which finally saves computing time and memory.

In cases of strong conductivity contrasts, particularly near the electrodes, errors arise in the forward calculation that can only be diminished by refinement techniques as the so-called h-method (finer discretization) or the p-method (higher polynomial basis functions). In order to achieve high speed and low memory consumption, the refinement has to be carried out adaptively such that only regions of weak approximation are locally refined. Another possibility of improving the forward calculation is a grid hierarchy used for multigrid solvers or preconditioners.

References


